An Unbiased Prodan Estimator

- stand density estimations applying the n-tree distance sampling -

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1. Introduction

Study is conducted on n-tree distance sampling - reliability in stand density estimations is evaluated and a new calculation approach denoted as a GM Estimator is proposed

2. N-tree distance sampling

- **Only one distance is measured from the plot center to the *n*th nearest tree**
- **Number of trees per plot is fixed and the circular plot area is a variable value**

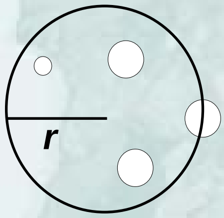

Figure 1. *The 4-tree sampling*

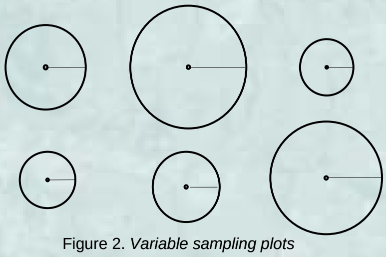

Figure 2. *Variable sampling plots*

3. Evaluated density estimators

- ***(n – 0.5) Prodan's assumption (Prodan, 1968)***

I. Generalized Prodan Estimator; calculating density applying an arithmetic mean (Payandeh and Ek, 1986)

$$N_i = \frac{A(n-0.5)}{\pi r_i^2}$$

$$\overline{N}_P = \frac{\sum_{i=1}^{m} N_i}{m}$$

A = unit area
n = fixed number of trees per plot
r = distance to the *n*th nearest tree
m = number of sampling plots

- ***(n-1)/n bias adjustment* (Moore, 1954)**

II. Unbiased Maximum Likelihood Estimator - Unbiased MLE; (Pollard, 1971). *Jonsson et al. (1992) termed it as the "DENSITY-ADAPTED METHOD"*

$$\overline{N}_{MLE} = \left(\frac{mn-1}{\pi \sum_{i=1}^{m} r_i^2} \right) A$$

III. Ratio of Means Estimator; (Payandeh and Ek, 1986). *Lessard et al. (1994), (1995), (2002); Lynch and Rusydi (1999) and Lynch and Wittwer (2003) term it as the "N-TREE DISTANCE SAMPLING METHOD"*

$$\overline{N}_{RME} = \frac{A(n-1)}{\pi m} \sum_{i=1}^{m} \frac{1}{r_i^2}$$

4. GM Estimator

Prodan (1968) proposed an approach in estimations of forest stand basal area assuming that one sampling plot considers of n-0.5 trees. Plot areas are variable in n-tree distance sampling and the squared mean will cause a noticeable bias. Therefore, a new calculation approach is proposed and here denoted as a *GM Estimator*, estimating stand density applying the geometric mean to the *(n-0.5) Prodan's assumption*.

$$\overline{N}_{GM} = \sqrt[m]{N_1 N_2 N_3 \ldots N_m}$$

5. Research methodology

Real forest stands are simulated and sampling performed in a GIS. Six point populations are created: four nearly random, one regular and one aggregated. It is assumed that each point represents a tree and each point population represents a forest stand.
Field measurements are conducted at 6.48ha of 98 years old Cryptomeria japonica plantation at the Tokyo University Forest in Chiba-Japan

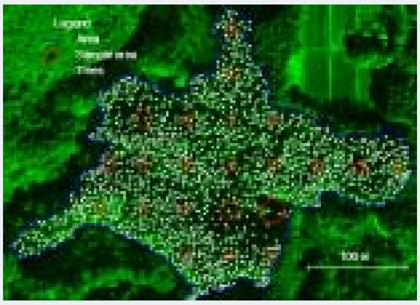

Figure 3. *Simulated forest stand applying the 4-tree sampling; mapped tree positions - Cryptomeria japonica plantation at the Tokyo University Forest in Chiba (5.03 ha - 330 trees/ha)*

Photo: Measured Cryptomeria plantation

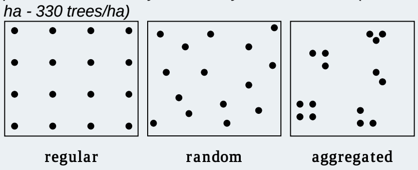

Figure 4. *Types of usually considered spatial distributions in ecological studies*

6. Results

The relative performance of each evaluated estimator (*Unbiased MLE*, *Ratio of Means Estimator* and *Generalized Prodan Estimator*) and proposed *GM Estimator* was compared on the basis of its relative error expressed by following equation:

Relative error = [(estimated density – true density) x 100] / true density

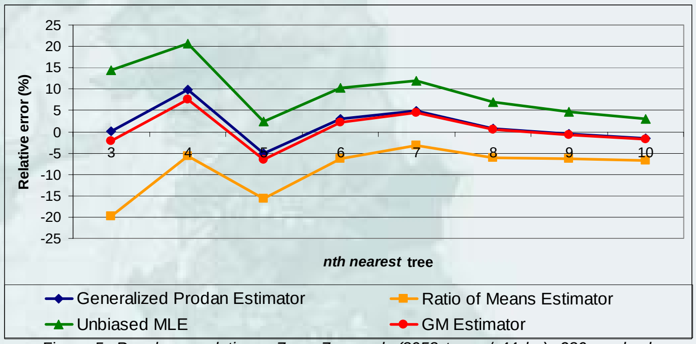

Figure 5. *Regular population – 7m x 7m mesh (8958 trees / 44 ha); 280 randomly distributed sampling plots*

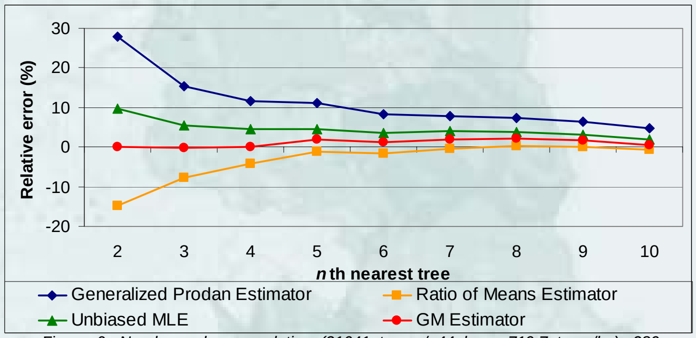

Figure 6. *Nearly random population (31641 trees / 44 ha – 719.7 trees/ha); 280 randomly distributed sampling plots*

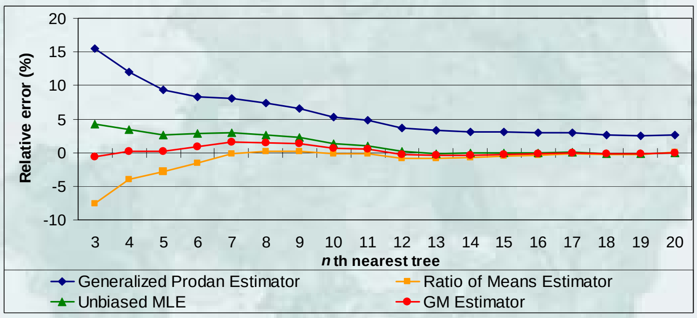

Figure 7. *Nearly random population (31641 trees / 44 ha – 719.7 trees/ha); 1190 sampling plots – cluster sampling procedure*

7. Conclusion

- Averaging variable circular plot areas the squared mean will cause a bias; with higher variation, the bias is higher.
- Bias can be removed calculating density using a geometric mean and the *0.5 Prodan`s assumption* – The GM Estimator
- Compared to evaluated n-tree distance sampling methods, applying smaller n-tree values the *GM Estimator* performed the most accurate density estimations. The *GM Estimator* can be regarded as a reliable density estimator in forest plantations and even-aged forests.
- **Any n-tree value can be applied to a forest inventory**; **the spatial distribution**, the size of a measured population, the sample size, measurement tools and techniques are crucial factors to set the n-tree value.
- Increasing the n-tree value an influence of a spatial distribution will become negligible
- Estimations with the n-tree distance sampling may be influenced by the edge effect and a considerable care is necessary especially if sampling small areas. Stratification, which is also recommended if applying other known statistical methods, has to be conducted in order to increase accuracy of estimations.

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Background image:
Map of the Tokyo University Forest in Chiba, Japan; *KOKUDOCHIZU co.,Ltd.*